

UNCLASSIFIED

Defense Technical Information Center
Compilation Part Notice

ADP013213

TITLE: Theory of Magnetophonon Resonance in Quantum Wells. Tilted Magnetic Field

DISTRIBUTION: Approved for public release, distribution unlimited
Availability: Hard copy only.

This paper is part of the following report:

TITLE: Nanostructures: Physics and Technology International Symposium [9th], St. Petersburg, Russia, June 18-22, 2001 Proceedings

To order the complete compilation report, use: ADA408025

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:

ADP013147 thru ADP013308

UNCLASSIFIED

Theory of magnetophonon resonance in quantum wells. Tilted magnetic field

V. V. Afonin[†], V. L. Gurevich[†] and R. Laiho[§]

[†] Ioffe Physico-Technical Institute, St Petersburg, Russia

[§] Wihuri Physical Laboratory, Department of Physics, University of Turku,
FIN-20014 Turku, Finland

Abstract. We develop a theory of magnetophonon resonance (MPR) in quantum wells in a tilted magnetic field. We explain the existence of two peaks of MPR in its angular dependence that may be very sharp. A relation between the MPR amplitude in the perpendicular magnetic field and its θ -dependence in a tilted field is discussed. We come to conclusion that the θ -dependence of MPR can give valuable information concerning the optic phonon damping and the electron-phonon interaction in quantum wells.

Magnetophonon resonance (MPR) is the first *internal* resonance in solids that has been predicted theoretically and subsequently observed experimentally (see the review paper [1]). The resonant condition is met every time when the limiting frequency of an optical phonon equals the cyclotron frequency of an electron, $\omega_B = eB/mc$, times some small integer, \mathcal{N} . Since its theoretical prediction and subsequent experimental discovery MPR has become a powerful tool to investigate the electron spectra in semiconductors. The magnetophonon resonance in quantum wells has been investigated since the pioneering paper by Tsui *et al.*. The most detailed experimental investigation of the phenomenon has been done by Nicholas with co-workers (see the review paper [2] and the references therein).

There are two main groups of such experiments. The first group deals with the MPR in the perpendicular (to the plane of 2DEG) magnetic field. The main features of this case are (i) the fact that the resonance is determined by the transverse optic frequency ω_t (rather than the longitudinal frequency ω_l) and (ii) a rather narrow interval of electron concentrations where the MPR is observable. The second group concerns with the experiments in magnetic field tilted at an angle θ to the perpendicular. For small values of θ the MPR is determined by ω_t . For slightly larger values its amplitude sharply goes down within a narrow angular interval of the order of 10° . For even bigger values of θ there is another maximum, this time determined by ω_l [3].

In our paper [4] we give interpretation of the first group of experiments. Here we offer interpretation of the second group and show that the angular and concentration dependencies of the MPR amplitudes are deeply interrelated.

We assume that the well is so narrow that only one electron band of spatial quantization is filled. The magnetic field \mathbf{B} is assumed to be in the (y, z) -plane, the z -axis being perpendicular to the 2DEG, while the external electric field is oriented along the y -axis.

We choose the following gauge for the vector potential $\mathbf{A} = (-By \cos \theta + Bz \sin \theta, 0, 0)$ and assume a parabolic confining potential $m\omega_0^2 z^2/2$ where m is the effective mass. It is also assumed that $\hbar\omega_0 \gg \hbar\Omega, k_B T$ (where $\Omega = eB/mc$ while T is the temperature). This assumption permits to consider only the lowest miniband. This means that our problem differs from that in perpendicular magnetic field by replacement $B \rightarrow B \cos \theta$.

Using the method developed by Kubo *et al.* and applying the method of Ref. [3] one can get for the conductance averaged over cross section of the sample

$$\sigma_{xx} = \frac{1}{2k_B T} \left(\frac{c}{B \cos \theta} \right)^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^2 q}{(2\pi)^2} \int dz \int dz' \frac{q_y^2 N(\omega)}{1 - \exp(-\hbar\omega/k_B T)} \times [D_R(\mathbf{q}, -\omega) - \mathbf{D}_A(\mathbf{q}, -\omega)][\Pi_R(\mathbf{q}, \omega; \mathbf{z}', \mathbf{z}) - \Pi_A(\mathbf{q}, \omega; \mathbf{z}', \mathbf{z})]. \quad (1)$$

Here $D_{R,A}$ is the full polarization optic phonon propagator while $\Pi_{R,A}$ is the electron polarization operator.

$$\Pi_R^{(2)} = -2n_s \exp \left[-\frac{(qa_B)^2 \coth \alpha}{2 \cos \theta} \right] \sum_{\mathcal{N}=-\infty}^{\infty} \frac{\sinh \mathcal{N}\alpha}{\omega - \mathcal{N}\Omega \cos \theta + i\delta} I_{\mathcal{N}} \left(\frac{q^2 a_B^2}{2 \cos \theta \sinh \alpha} \right). \quad (2)$$

Here $I_{\mathcal{N}}$ is the modified Bessel function, $\alpha = \hbar\Omega \cos \theta / 2k_B T$, $a_B^2 = c\hbar/eB$, n_s is the 2D electron concentration. The polarization operator of Eq. (1) differs from $\Pi_R^{(2)}$ by the factor $\psi(z)\psi(z')$ due to the electron propagation along the z -axis. Here $\psi(z)$ is the wave function of the lowest level of the transverse quantization.

The zeroth-order phonon propagator (including the Frölich electron-phonon interaction) is

$$D_R^{(0)}(\omega, \mathbf{q}) = \frac{4\pi \mathbf{e}^2}{\mathbf{q}^2 \varepsilon(\omega + i\Gamma)}, \quad \varepsilon(\omega) = \varepsilon_{\infty} \frac{\omega_1^2 - \omega^2}{\omega_1^2 - \omega^2} \quad (3)$$

where ε_{∞} is the lattice dielectric susceptibility at $\omega \rightarrow \infty$ while Γ is the phonon damping. As in Ref. [3], we assume that it is determined by the lattice anharmonicity. Further on we assume that one can neglect the difference between the lattice properties within and outside the well. Both these assumptions should not affect the qualitative results of the theory.

Eq. (2) shows that the electron-phonon interaction cannot be treated within the perturbation theory. The point is that the higher orders of the perturbation theory (without regard of the electron damping Γ_e) give powers of an extra factor $1/(\omega - \mathcal{N}\Omega \cos \theta + i\delta)$. Therefore, as is shown in Ref. [3], the phonon Green function includes a sum of chains of loop diagrams. Physically this means taking into account the screening of the phonon polarization potential by the conduction electrons. Thus in 2D case in a resonance the screening can be very important.

One should observe the following essential point exploited in Ref. [3]. Both ends of the chain *should be ordinary phonon lines without addition of any Coulomb interaction lines*. This is due to the fact that the electron-electron (e-e) interaction conserves the electron quasimomentum.

Thus the oscillatory part of σ_{xx} near the \mathcal{N} th MPR (without regard of the electron damping) is given by

$$\Delta\sigma_{xx} = \frac{2n_s c^2 \hbar^2 \mathcal{J}_{\mathcal{N}}}{\varepsilon_{\infty}^2 k_B T B^2 \cos^2 \theta} \frac{N(\omega_t) \sinh(\hbar\omega_t/2k_B T)}{1 - \exp(-\hbar\omega_t/k_B T)} \text{Im } \varepsilon_R \Big|_{\omega = \mathcal{N}\Omega \cos \theta + i\Gamma} \quad (4)$$

where

$$\mathcal{J}_{\mathcal{N}} = 2\pi e^2 \int \frac{d^2 q}{(2\pi)^2} I_{\mathcal{N}} \left(\frac{a_B^2 q^2}{2 \cos \theta \sinh \alpha} \right) \exp \left(-\frac{a_B^2 q^2 \coth \alpha}{2 \cos \theta} \right). \quad (5)$$

As $\text{Im } \varepsilon$ has a singularity at $\omega = \omega_t$, $\Delta\sigma_{xx}$ exhibits the MPR's at $\mathcal{N}\Omega \cos \theta = \omega_t$. Physically this is due to the fact that the e-e interaction without regard of the damping is infinitely strong in the resonance.

Further on we will treat the case $\hbar\Omega \ll k_B T$ that corresponds to a most usual experimental situation. Then

$$\mathcal{J}_{\mathcal{N}} \approx \frac{2}{\sqrt{\pi}} \frac{e^2}{a_B^3} \sqrt{\frac{k_B T}{\hbar\Omega}} \cos \theta. \quad (6)$$

One can see that the integral (4) $\mathcal{J}_{\mathcal{N}}$ is dominated by $q \approx q_T = \sqrt{2mk_B T}/\hbar$.

In order to explain the sharp angular dependence of the MPR's one should take into account the electron damping. As a result, for small electron concentrations [3] (or, for large values of θ) one can neglect the e-e interaction at the frequencies ω near ω_t and the MPR at this phonon frequency disappears. This is why we take into account the electron damping Γ_e . The characteristic angle θ_t of the sharpest angular dependence of the MPR maximum can be determined experimentally as a minimum of the derivative of the MPR amplitude over θ .

We assume that $\Gamma_e \ll \Omega \cos \theta$. The electron Green function in magnetic field has been investigated by Ando and Uemura for Γ_e determined by the elastic scattering. They have shown that the electron Green function has a non-Lorentzian form with the characteristic width Γ_e given by $\Gamma_e^2 = \Omega \cos \theta / 2\pi\tau$ where τ is the relaxation time for $B = 0$ obtained by assuming the same scatterers as for finite B .

For the order-of-magnitude estimates it will be sufficient to use the Lorentzian form of $\Pi_R^{(2)}(\omega, \mathbf{q})$. Moreover, in the resonance approximation one should retain only the resonant term of all the series for $\Pi_R^{(2)}(\omega, \mathbf{q})$

$$\Pi_R^{(2)}(\omega, \mathbf{q}) = -\frac{\mathcal{R}_{\mathcal{N}}(\omega, \mathbf{q})}{\omega - \mathcal{N}\Omega \cos \theta + i\Gamma_e}. \quad (7)$$

where $\mathcal{R}_{\mathcal{N}}$ is the residue at the pole $\omega = \mathcal{N}\Omega \cos \theta - i\Gamma_e$. Calculating $\Delta\sigma_{xx}$ one can evaluate the integral over frequency taking the residues in the poles $\omega = \mathcal{N}\Omega \cos \theta \pm i\Gamma_e$. This results in replacement of $\text{Im } \varepsilon_R$ under the integral by

$$\Delta \equiv \frac{1}{\pi} \frac{\text{Im } \varepsilon_A^{-1} + 2\gamma}{(2\gamma + \text{Im } \varepsilon_A^{-1})^2 + (\text{Re } \varepsilon_A^{-1})^2} \quad (8)$$

where $\gamma = \Gamma_e/\bar{\omega}$; $\bar{\omega} = 2\pi e^2/q\mathcal{R}_{\mathcal{N}}(\mathcal{N}\Omega \cos \theta, \mathbf{q})$ while ε_A is calculated at $\omega = \mathcal{N}\Omega \cos \theta + i(\Gamma + \Gamma_e)$. The angle θ_t is given by the condition $2\gamma = \varepsilon_A^{-1}$. As the integral in (4) is dominated by $q = q_T$ this condition should be fulfilled for $q = q_T$ and has the form

$$\frac{1}{1 + \Gamma/\Gamma_e(\theta_t)} = \frac{n_s}{n_1} \cos \theta_t. \quad (9)$$

Here and henceforth we assume that $\Gamma, \Gamma_e \ll \omega_l - \omega_t \ll \omega_l$ while

$$\frac{1}{n_1} = \frac{e^2 \Omega^{1/2} \omega_t}{\varepsilon_{\infty} \hbar a_B q_T^2 T^{3/2} (\omega_l - \omega_t)}. \quad (10)$$

Eq. (9) establishes a correspondence between the low concentration dependence of the MPR amplitude for perpendicular \mathbf{B} [3] and its angular dependence in a tilted field \mathbf{B} for a fixed concentration. Indeed, the sharpest dependence of MPR amplitude on n_s as well

as on θ comes from the resonant factor Eq. (8). In particular, for $\Gamma/\Gamma_e \ll 1$ the MPR amplitude is determined by the effective concentration $n_s \cos \theta$. Thus according to Eq. (9) the decrease of the MPR amplitude for $\theta = 0$ when n_s goes down and its decrease when θ goes up are interrelated.

For further enhancement of the angle θ the resonance at $\mathcal{N}\Omega \cos \theta = \omega_l$ disappears. As soon as the condition $2\gamma = \varepsilon_A^{-1}$ is satisfied for $\omega = \omega_l + i\Gamma + i\Gamma_e$ direct application of the perturbation theory is permissible [3] as the screening ceases to play any role. Then we have a resonance at $\mathcal{N}\Omega \cos \theta = \omega_l$. The angle corresponding to the amplitude maximum will be denoted by θ_l . Due to the strong dispersion of $\varepsilon(\omega)$ the angles θ_t and θ_l can be discerned on experiment. Thus the equation $2\gamma = \varepsilon_A^{-1}$ for calculation of θ_t and θ_l has the same form for frequencies ω_t and ω_l respectively. As a result, we have

$$\sqrt{\frac{\cos \theta_l}{\cos \theta_t}} = \frac{\text{Im } \varepsilon^{-1}(\omega_t + i\Gamma + i\Gamma_e, \theta_t)}{\text{Im } \varepsilon^{-1}(\omega_l + i\Gamma + i\Gamma_e, \theta_l)}. \quad (11)$$

The dependence of ε^{-1} on θ is due to the θ -dependence of Γ_e . We find a reasonable correspondence between the experiment and this theory.

To summarize, we stress that the interpretation of behavior of the MPR in a tilted magnetic field has been a long-standing problem [2]. Two types of resonant maxima have been discovered on experiment. They may be called the ω_t - and ω_l -resonances as their positions are determined by the frequencies ω_t and ω_l respectively. We have determined the angular intervals where both types of resonance exist. We have found that sharp decrease of the ω_t -resonance amplitudes is due to the sharp angular dependence of the screening. The ω_l -resonance is analogous to the 3D MPR as there the screening plays no role. Therefore this resonance is suitable for investigation of the electron spectrum in the quantum wells. We wish to emphasize that its experimental investigation in the perpendicular magnetic field should be very difficult as it would demand very low electron concentrations [3]. For bigger electron concentrations one can expect an enhancement of the MPR amplitudes for large values of θ .

Acknowledgements

V.V.A. and V.L.G. are grateful to the Wihuri Foundation as well as to the Russian Foundation for Basic Research, grant No 00-15-96748 for financial support.

References

- [1] Yu. A. Firsov, V. L. Gurevich, R. V. Parfeniev and I. M. Tsidil'kovskii, *Magnetophonon Resonance*. In: *Landau Level Spectroscopy*, ed. G. Landwehr and E. I. Rashba, Elsevier, 1991.
- [2] R. J. Nicholas, *Two-dimensional Magnetophonon Effect*. In: *Landau Level Spectroscopy*, ed. G. Landwehr and E. I. Rashba, Elsevier, 1991, p. 778.
- [3] V. V. Afonin, V. L. Gurevich and R. Laiho, *Phys. Rev. B* **62**, 15 913 (2000).